FREE NOTES · NCERT + JEE LEVEL

Linear Inequalities

Inequalities aur unke rules, ek variable wali linear inequalities solve karna, number line pe dikhana, do variables wali inequalities ka graph aur systems of inequalities.

01 Inequality kya hai
02 Solve karne ke rules
03 Ek variable wali linear inequalities
04 Number line pe solution
05 Do variables wali linear inequalities
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01 Inequality kya hai

Do expressions jo $<$, $>$, $\leq$ ya $\geq$ se jude hon. Examples: $3x - 5 < 7$ (linear, ek variable), $2x + 3y \geq 6$ (linear, do variables), $x^2 + 1 > 0$ (quadratic). **Strict** ($<$, $>$) vs **slack** ($\leq$, $\geq$).

02 Solve karne ke rules

DO RULES

1. Dono taraf same number jodo ya ghatao: sign wahi rehta.
2. Dono taraf same **positive** number se guna/bhaag: sign wahi. **Negative** number se: sign **ulat** jaata ($<$ ban jaata $>$).

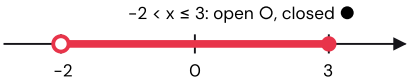
Example: $-3x < 9 \Rightarrow x > -3$ (-3 se bhaag diya, sign palta).

03 Ek variable wali linear inequalities

Equation ki tarah solve karo, par negative se guna/bhaag pe sign ka dhyan. Answer allowed set pe depend: $30x < 200$ me agar $x \in N$: {1, 2, ..., 6}; $x \in Z$: {..., 5, 6}; $x \in R$: $x < 20/3$.

> **Example:** $(3x - 4)/2 \geq (x + 1)/4 - 1 \Rightarrow 2(3x - 4) \geq (x + 1) - 4 \Rightarrow 6x - 8 \geq x - 3 \Rightarrow 5x \geq 5 \Rightarrow \mathbf{x \geq 1}$, yaani $[1, \infty)$.

04 Number line pe solution



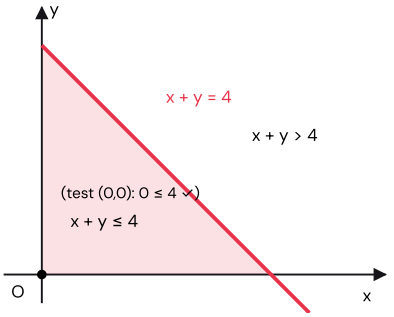
Khokhla gola = shamil nahi (<, >); bhara = shamil (≤, ≥)

Double inequality: $-8 \leq 5x - 3 < 7 \Rightarrow -5 \leq 5x < 10 \Rightarrow \mathbf{-1 \leq x < 2}$, yaani $[-1, 2)$.

05 Do variables wali linear inequalities

Line $ax + by = c$ plane ko do **half-planes** me baant-ti hai.

1. Line banao: </> ke liye **dotted**, ≤/≥ ke liye **solid**.
2. Line pe na ho aisa test point lo, aksar (0, 0).
3. Agar wo inequality satisfy kare to uski side shade karo; warna doosri side.
4. Line (0, 0) se guzre to (1, 0) ya (0, 1) test karo.



x + y ≤ 4 ka solution region

06 Systems of inequalities

Har inequality ka graph banao; **common shaded region** hi solution. x ≥ 0, y ≥ 0 pehle quadrant me rakhte. Koi common region na ho to koi solution nahi.

EXAMPLE

x + y ≥ 5 aur x − y ≤ 3: dono lines banao; x + y = 5 ke upar shade (kyunki (0,0) fail) aur x − y = 3 ki (0,0) wali side. Overlap hi answer. Class 12 me yahi Linear Programming banta.

07 Word problems

- "Kam se kam" ⇒ ≥, "zyada se zyada" ⇒ ≤, "se zyada" ⇒ >, "se kam" ⇒ <.
- Marks example: Ravi ke do tests me 70 aur 75. Teesre me kam se kam kitne ki average ≥ 60 ho: (70 + 75 + x)/3 ≥ 60 ⇒ x ≥ 35.
- Lagatar odd numbers dono > 10, jod < 40: x > 10, x + (x + 2) < 40 ⇒ 10 < x < 19 ⇒ (11,13), (13,15), (15,17), (17,19).

08 JEE Extra + Tricks

JEE LEVEL

- Negative number se guna/bhaag ⇒ inequality ulti.
- $|x| < a \Leftrightarrow -a < x < a$; $|x| > a \Leftrightarrow x < -a$ ya $x > a$.
- AM ≥ GM ≥ HM (sab barabar pe equality); min/max ke liye: $x > 0$ pe $x + 1/x \geq 2$.
- $f(x)/g(x) > 0$ me wavy curve; unknown sign wale $g(x)$ se kabhi guna mat karo.
- Graph wale sawaal: kaunsi taraf shade, (0, 0) daalkar check karo.

09 Solved examples

Q1. $3(x - 1) \leq 2(x - 3)$ solve karo.

$3x - 3 \leq 2x - 6 \Rightarrow$ **$x \leq -3$**, $(-\infty, -3]$.

Q2. $-5 \leq (5 - 3x)/2 \leq 8$ solve karo.

$-10 \leq 5 - 3x \leq 16 \Rightarrow -15 \leq -3x \leq 11 \Rightarrow 5 \geq x \geq -11/3 \Rightarrow$ **$[-11/3, 5]$**.

Q3. 600 L 12% acid me kitna 30% acid milayein ki result 15% aur 18% ke beech ho?

$15 < (0.3x + 72)/(600 + x) \times 100 < 18 \Rightarrow$ **$120 < x < 300$** litre.

10 Common mistakes

- Negative se guna karke sign na palatna.
- Strict inequality ke liye solid line banana.
- Line ke upar wala point test karna.
- Dhyan na dena ki x natural, integer ya real hona chahiye.

11 Practice questions

1. $4x + 3 < 5x + 7$ solve karo.
2. $x/3 > x/2 + 1$ solve karo.
3. $3 \le 4 - 7x < 18$ solve karo (fraction me).
4. $2x - y < 4$ ke liye (0, 0) kis side?

ANSWERS

1) $x > -4$ · 2) $x < -6$ · 3) $-2 < x \le 1/7$ · 4) Satisfy karta ($0 - 0 < 4$), to origin wali side shade

12 Quick revision: exam se 5 minute pehle

RULES

- Negative se × ya ÷ ⇒ palto.
- Jodna/ghatana free.

NUMBER LINE

- $<$, $>$ ke liye ○.
- $\le$, $\ge$ ke liye ●.

DO VARIABLES

- Dotted vs solid line.
- (0, 0) test.

SYSTEMS

- Common region = answer.
- Overlap nahi ⇒ solution nahi.

Koi doubt? padhaiadda.netlify.app pe AI Doubt Solver se poocho, ya is chapter ka mock test do.