FREE NOTES · NCERT + JEE LEVEL

Linear Inequalities

Inequalities and their rules, solving linear inequalities in one variable, showing solutions on the number line, graphing inequalities in two variables and systems of inequalities.

01 What is an inequality
02 Rules for solving
03 Linear inequalities in one variable
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01 What is an inequality

Two expressions joined by $<$, $>$, $\leq$ or $\geq$. Examples: $3x - 5 < 7$ (linear, one variable), $2x + 3y \geq 6$ (linear, two variables), $x^2 + 1 > 0$ (quadratic). **Strict** ($<$, $>$) vs **slack** ($\leq$, $\geq$).

02 Rules for solving

TWO RULES

1. Add or subtract the same number on both sides: the sign stays.
2. Multiply or divide both sides by the same **positive** number: the sign stays. By a **negative** number: the sign **reverses** ($<$ becomes $>$).

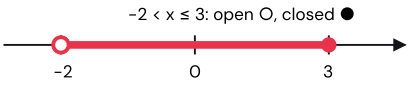
Example: $-3x < 9 \Rightarrow x > -3$ (divided by -3, sign flipped).

03 Linear inequalities in one variable

Treat like an equation, but watch the sign when multiplying/dividing by negatives. The answer depends on the allowed set: for $30x < 200$, if $x \in N$: {1, 2, ..., 6}; if $x \in Z$: {..., 5, 6}; if $x \in R$: $x < 20/3$.

Example: $(3x - 4)/2 \geq (x + 1)/4 - 1 \Rightarrow 2(3x - 4) \geq (x + 1) - 4 \Rightarrow 6x - 8 \geq x - 3 \Rightarrow 5x \geq 5 \Rightarrow \mathbf{x \geq 1}$, i.e. $[1, \infty)$.

04 Showing solutions on the number line



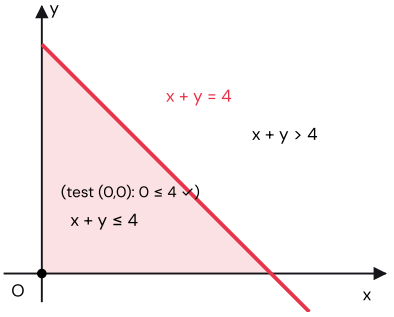
Hollow circle = not included (<, >); filled = included (≤, ≥)

Double inequality: $-8 \leq 5x - 3 < 7 \Rightarrow -5 \leq 5x < 10 \Rightarrow \mathbf{-1 \leq x < 2}$, i.e. $[-1, 2)$.

05 Linear inequalities in two variables

The line $ax + by = c$ divides the plane into two **half-planes**.

1. Draw the line: **dotted** for </>, **solid** for ≤/≥.
2. Pick a test point not on the line, usually (0, 0).
3. If it satisfies the inequality, shade its side; otherwise the other side.
4. If the line passes through (0, 0), test (1, 0) or (0, 1).



Solution region of $x + y \leq 4$

06 Systems of inequalities

Graph each inequality; the **common shaded region** is the solution. $x \geq 0$, $y \geq 0$ keep you in the first quadrant. If no region is common, the system has no solution.

EXAMPLE

$x + y \geq 5$ and $x - y \leq 3$: draw both lines; shade above $x + y = 5$ (since (0,0) fails) and the side of $x - y = 3$ containing (0,0). The overlap is the answer. This idea becomes Linear Programming in Class 12.

07 Word problems

- "At least" ⇒ ≥, "at most" ⇒ ≤, "more than" ⇒ >, "less than" ⇒ <.
- Marks example: Ravi got 70 and 75 in two tests. Minimum in the third for an average of at least 60: $(70 + 75 + x)/3 \geq 60 \Rightarrow x \geq 35$.
- Consecutive odd numbers both > 10, sum < 40: $x > 10$, $x + (x + 2) < 40 \Rightarrow 10 < x < 19 \Rightarrow$ (11,13), (13,15), (15,17), (17,19).

08 JEE Extra + Tricks

JEE LEVEL

- Multiplying/dividing an inequality by a negative number reverses it.
- $|x| < a \Leftrightarrow -a < x < a$; $|x| > a \Leftrightarrow x < -a$ or $x > a$.
- AM ≥ GM ≥ HM (equality when all equal); used for min/max: $x + 1/x \geq 2$ for $x > 0$.
- For $f(x)/g(x) > 0$ use wavy curve; never multiply both sides by $g(x)$ of unknown sign.
- Graphical LP-type questions: test origin (0, 0) to decide shading side.

09 Solved examples

Q1. Solve $3(x - 1) \leq 2(x - 3)$.

$3x - 3 \leq 2x - 6 \Rightarrow \mathbf{x \leq -3}$, $(-\infty, -3]$.

Q2. Solve $-5 \leq (5 - 3x)/2 \leq 8$.

$-10 \leq 5 - 3x \leq 16 \Rightarrow -15 \leq -3x \leq 11 \Rightarrow 5 \geq x \geq -11/3 \Rightarrow$ **$[-11/3, 5]$**.

Q3. How much 30% acid must be added to 600 L of 12% acid so the result is between 15% and 18%?

$15 < (0.3x + 72)/(600 + x) \times 100 < 18 \Rightarrow$ **$120 < x < 300$** litres.

10 Common mistakes

- Not flipping the sign after multiplying by a negative number.
- Drawing a solid line for a strict inequality.
- Testing a point that lies on the line.
- Ignoring whether x must be natural, integer or real.

11 Practice questions

1. Solve $4x + 3 < 5x + 7$.
2. Solve $x/3 > x/2 + 1$.
3. Solve $3 \le 4 - 7x < 18$ (in fraction form).
4. Which side of $2x - y = 4$ does (0, 0) lie on, for $2x - y < 4$?

ANSWERS

1) $x > -4$ · 2) $x < -6$ · 3) $-2 < x \le 1/7$ · 4) It satisfies it ($0 - 0 < 4$), so shade the origin side

12 Quick revision: 5 minutes before the exam

RULES

- × or ÷ by negative ⇒ flip.
- Add/subtract freely.

NUMBER LINE

- ○ for <, >.
- ● for ≤, ≥.

TWO VARIABLES

- Dotted vs solid line.
- Test (0, 0).

SYSTEMS

- Common region = answer.
- No overlap ⇒ no solution.

Have a doubt? Ask the AI Doubt Solver at padhaiadda.netlify.app, or take the mock test on this chapter.