FREE NOTES · NCERT + JEE LEVEL

Complex Numbers & Quadratic Equations

Imaginary unit aur i ki powers, complex numbers aur unka algebra, conjugate aur modulus, Argand plane aur polar form, aur complex roots wali quadratic equations.

01 Iota (i) & uski powers
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01 Iota (i) & uski powers

$x^2 + 1 = 0$ ka koi real root nahi. Isliye **$i = \sqrt{-1}$** define kiya, $i^2 = -1$.

$i^1 = i,\ i^2 = -1,\ i^3 = -i,\ i^4 = 1$ (phir repeat)

$i^{4k} = 1$; i^n ke liye n ko 4 se bhaag do, remainder use karo.

$i + i^2 + i^3 + i^4 = 0$

$\sqrt{a}\cdot\sqrt{b} = \sqrt{ab}$ tabhi jab a, b dono negative na hon: $\sqrt{-4}\cdot\sqrt{-9} = 2i\cdot 3i = -6$, $\sqrt{36}$ nahi.

02 Complex numbers

$z = a + ib$, $a, b \in R$. **Re(z) = a**, **Im(z) = b**. a = 0 ho to purely imaginary; b = 0 ho to real. Do complex numbers barabar tab jab real parts aur imaginary parts alag alag barabar hon. Complex numbers me ">" ya "<" nahi hota.

03 Complex numbers ka algebra

$(a + ib) \pm (c + id) = (a \pm c) + i(b \pm d)$

$(a + ib)(c + id) = (ac - bd) + i(ad + bc)$

$1/z = \bar{z}/|z|^2$ (multiplicative inverse); $z_1/z_2 = z_1\,\bar{z}_2/|z_2|^2$

$(z_1 + z_2)^2 = z_1^2 + 2z_1z_2 + z_2^2$ jaise identities chalti hain.

04 Conjugate & modulus

- **Conjugate** $\bar{z} = a - ib$. $z + \bar{z} = 2\,\mathrm{Re}\,z$, $z - \bar{z} = 2i\,\mathrm{Im}\,z$, $z\,\bar{z} = a^2 + b^2$.
- **Modulus** $|z| = \sqrt{(a^2 + b^2)}$ (origin se doori).

$$|z_1 z_2| = |z_1||z_2|,\ |z_1/z_2| = |z_1|/|z_2|,\ |\bar{z}| = |z|$$

$$\overline{(z_1 \pm z_2)} = \bar{z}_1 \pm \bar{z}_2;\ \overline{(z_1 z_2)} = \bar{z}_1\,\bar{z}_2$$

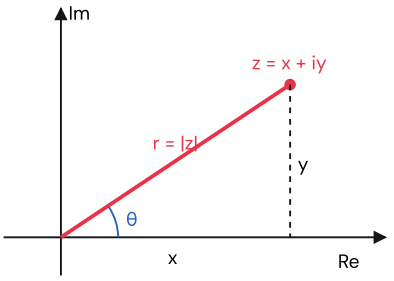
$$|z_1 + z_2| \le |z_1| + |z_2| \text{ (triangle inequality)}$$

05 Argand plane & polar form

$z = x + iy$ matlab point (x, y). x-axis = real axis, y-axis = imaginary axis.

$$z = r(\cos\theta + i\sin\theta)$$

$r = |z|$, $\theta = \arg z$ (argument). **Principal argument** $-\pi < \theta \le \pi$. Pehle $\alpha = \tan^{-1}|y/x|$ nikalo, phir: Q1 $\theta = \alpha$, Q2 $\theta = \pi - \alpha$, Q3 $\theta = -(\pi - \alpha)$, Q4 $\theta = -\alpha$.



Argand plane: modulus aur argument

06 Quadratic equations

$$ax^2 + bx + c = 0,\ D = b^2 - 4ac$$

$$x = (-b \pm \sqrt{D})/2a$$

$D > 0$: real alag; $D = 0$: real barabar; $D < 0$: complex conjugate pair $(-b \pm i\sqrt{(-D)})/2a$. Roots ka jod $= -b/a$, guna $= c/a$. Real coefficients ho to complex roots hamesha conjugate pair me aate.

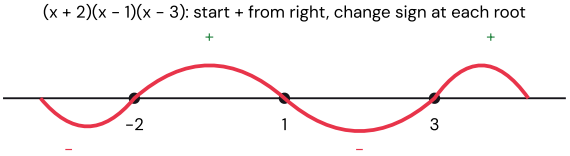
07 Formula sheet

i^4	1
$\lvert z\rvert$	$\sqrt{(a^2 + b^2)}$
$z\,\bar{z}$	$\lvert z\rvert^2$
z^{-1}	$\bar{z}/\lvert z\rvert^2$
Polar	$r(\cos\theta + i\sin\theta)$
Roots	$(-b \pm \sqrt{D})/2a$

08 JEE Extra + Tricks

JEE LEVEL

- Cube roots of unity: 1, ω, ω^2; $1 + \omega + \omega^2 = 0$, $\omega^3 = 1$, $\omega = (-1 + i\sqrt{3})/2$.
- De Moivre: $(\cos\theta + i\sin\theta)^n = \cos n\theta + i \sin n\theta$; nth roots of unity ka jod 0.
- $|z_1 + z_2| \le |z_1| + |z_2|$; $|z - z_0| = r$ circle; $|z - z_1| = |z - z_2|$ perpendicular bisector.
- Quadratic: $D \ge 0 \Rightarrow$ real roots; dono positive $\Rightarrow D \ge 0$, $-b/a > 0$, $c/a > 0$. Common root: $(c_1a_2 - c_2a_1)^2 = (a_1b_2 - a_2b_1)(b_1c_2 - b_2c_1)$.
- Inequalities ke liye **wavy curve method** (figure): roots mark karo, daayein se + shuru, sign badlo (even power wale root pe nahi).
- Roots ki location: $a \cdot f(k) < 0 \Rightarrow$ k roots ke beech.



Wavy curve method

09 Solved examples

Q1. i^{-39} nikalo.

$i^{-39} = 1/i^{39} = 1/i^3 = 1/(-i) =$ **i**.

Q2. $(5 - 3i)^3$ ko a + ib me likho.

$125 - 225i + 135i^2 - 27i^3 = 125 - 225i - 135 + 27i =$ **$-10 - 198i$**.

Q3. $2 - 3i$ ka multiplicative inverse.

$(2 + 3i)/13 =$ **$2/13 + 3i/13$**.

Q4. $-1 + i$ ka polar form.

$r = \sqrt{2}$, Q2 $\Rightarrow \theta = \pi - \pi/4 = 3\pi/4 \Rightarrow$ **$\sqrt{2}(\cos 3\pi/4 + i \sin 3\pi/4)$**.

Q5. $x^2 + x + 1 = 0$ solve karo.

$D = -3 \Rightarrow x = (-1 \pm i\sqrt{3})/2$.

10 Common mistakes

- $\sqrt{(-4)}\cdot\sqrt{(-9)} = \sqrt{36} = 6$ (galat; -6 hai).
- Quadrant check kiye bina $\arg z = \tan^{-1}(y/x)$ lena.
- $3 + 2i > 1 + i$ kehna (complex numbers me order nahi).
- $3 - 4i$ ka conjugate $-3 + 4i$ likhna ($3 + 4i$ hai).

11 Practice questions

1. $1 + i^2 + i^4 + i^6 + \ldots + i^{20}$ ki value?
2. $|(1 + i)/(1 - i)|$ nikalo.
3. $(x + iy)(2 - 3i) = 4 + i$ ho to real x, y?
4. $-\sqrt{3} - i$ ka principal argument?
5. $2x^2 + x + 1 = 0$ solve karo.

ANSWERS

1) 1 · 2) 1 · 3) $x = 5/13, y = 14/13$ · 4) $-5\pi/6$ · 5) $(-1 \pm i\sqrt{7})/4$

12 Quick revision: exam se 5 minute pehle

I KI POWERS

- i, −1, −i, 1 cycle.
- Lagatar 4 ka jod = 0.

ALGEBRA

- $z\,\bar{z} = |z|^2$.
- $1/z = \bar{z}/|z|^2$.
- $|z_1 z_2| = |z_1||z_2|$.

POLAR

- $r = |z|$, θ $(-\pi, \pi]$ me.
- θ ke liye quadrant check.

QUADRATIC

- $D < 0 \Rightarrow$ conjugate roots.
- Jod $-b/a$, guna c/a.

Koi doubt? padhaiadda.netlify.app pe AI Doubt Solver se poocho, ya is chapter ka mock test do.