FREE NOTES · NCERT + JEE LEVEL

Complex Numbers & Quadratic Equations

Imaginary unit and powers of i, complex numbers and their algebra, conjugate and modulus, Argand plane and polar form, and quadratic equations with complex roots.

01 Iota (i) & its powers
02 Complex numbers
03 Algebra of complex numbers
04 Conjugate & modulus
05 Argand plane & polar form
06 Quadratic equations
07 Formula sheet
08 JEE Extra + Tricks
09 Solved examples
10 Common mistakes
11 Practice questions
12 Quick revision

01 Iota (i) & its powers

$x^2 + 1 = 0$ has no real root. So we define **$i = \sqrt{-1}$**, $i^2 = -1$.

$i^1 = i, i^2 = -1, i^3 = -i, i^4 = 1$ (then repeats)

$i^{4k} = 1$; for i^n, divide n by 4 and use the remainder.

$i + i^2 + i^3 + i^4 = 0$

$\sqrt{a}\cdot\sqrt{b} = \sqrt{ab}$ only if a, b are not both negative: $\sqrt{(-4)}\cdot\sqrt{(-9)} = 2i\cdot 3i = -6$, not $\sqrt{36}$.

02 Complex numbers

z = a + ib, a, b ∈ R. **Re(z) = a**, **Im(z) = b**. If a = 0, z is purely imaginary; if b = 0, z is real. Two complex numbers are equal if real parts and imaginary parts are equal separately. Complex numbers can't be ordered (no ">" or "<").

03 Algebra of complex numbers

$(a + ib) \pm (c + id) = (a \pm c) + i(b \pm d)$

$(a + ib)(c + id) = (ac - bd) + i(ad + bc)$

$1/z = \bar{z}/|z|^2$ (multiplicative inverse); $z_1/z_2 = z_1\,\bar{z}_2/|z_2|^2$

Identities like $(z_1 + z_2)^2 = z_1^2 + 2z_1z_2 + z_2^2$ still hold.

04 Conjugate & modulus

- **Conjugate** $\bar{z} = a - ib$. $z + \bar{z} = 2\,\mathrm{Re}\,z$, $z - \bar{z} = 2i\,\mathrm{Im}\,z$, $z\,\bar{z} = a^2 + b^2$.
- **Modulus** $|z| = \sqrt{(a^2 + b^2)}$ (distance from origin).

$$|z_1 z_2| = |z_1||z_2|,\ |z_1/z_2| = |z_1|/|z_2|,\ |\bar{z}| = |z|$$

$$\overline{(z_1 \pm z_2)} = \bar{z}_1 \pm \bar{z}_2;\ \overline{(z_1 z_2)} = \bar{z}_1\,\bar{z}_2$$

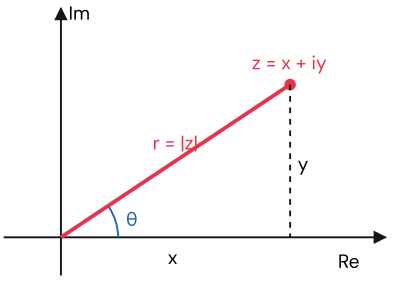
$$|z_1 + z_2| \le |z_1| + |z_2| \text{ (triangle inequality)}$$

05 Argand plane & polar form

$z = x + iy$ is the point (x, y). x-axis = real axis, y-axis = imaginary axis.

$$z = r(\cos\theta + i\sin\theta)$$

$r = |z|$, $\theta = \arg z$ (argument). **Principal argument** $-\pi < \theta \le \pi$. Find $\alpha = \tan^{-1}|y/x|$ then: Q1 $\theta = \alpha$, Q2 $\theta = \pi - \alpha$, Q3 $\theta = -(\pi - \alpha)$, Q4 $\theta = -\alpha$.



Argand plane: modulus and argument

06 Quadratic equations

$$ax^2 + bx + c = 0,\ D = b^2 - 4ac$$

$$x = (-b \pm \sqrt{D})/2a$$

$D > 0$: real distinct; $D = 0$: real equal; $D < 0$: complex conjugate pair $(-b \pm i\sqrt{(-D)})/2a$. Sum of roots $= -b/a$, product $= c/a$. With real coefficients, complex roots always come in conjugate pairs.

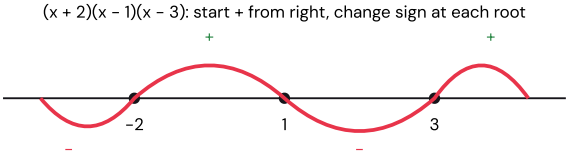
07 Formula sheet

i^4	1
$\lvert z\rvert$	$\sqrt{(a^2 + b^2)}$
$z\,\bar{z}$	$\lvert z\rvert^2$
z^{-1}	$\bar{z}/\lvert z\rvert^2$
Polar	$r(\cos\theta + i\sin\theta)$
Roots	$(-b \pm \sqrt{D})/2a$

08 JEE Extra + Tricks

JEE LEVEL

- Cube roots of unity: $1, \omega, \omega^2$; $1 + \omega + \omega^2 = 0$, $\omega^3 = 1$, $\omega = (-1 + i\sqrt{3})/2$.
- De Moivre: $(\cos\theta + i\sin\theta)^n = \cos n\theta + i \sin n\theta$; nth roots of unity sum to 0.
- $|z_1 + z_2| \le |z_1| + |z_2|$; $|z - z_0| = r$ is a circle; $|z - z_1| = |z - z_2|$ is the perpendicular bisector.
- Quadratic: roots real iff $D \ge 0$; both roots positive iff $D \ge 0$, $-b/a > 0$, $c/a > 0$. Common root condition: $(c_1a_2 - c_2a_1)^2 = (a_1b_2 - a_2b_1)(b_1c_2 - b_2c_1)$.
- **Wavy curve method** for inequalities (figure): mark roots, start + from the right, alternate signs (skip change at even-power roots).
- Location of roots: k lies between roots iff $a \cdot f(k) < 0$.



Wavy curve method

09 Solved examples

Q1. Find i^{-39}.

$i^{-39} = 1/i^{39} = 1/i^3 = 1/(-i) =$ **i**.

Q2. Express $(5 - 3i)^3$ in a + ib form.

$125 - 225i + 135i^2 - 27i^3 = 125 - 225i - 135 + 27i =$ **$-10 - 198i$**.

Q3. Find the multiplicative inverse of $2 - 3i$.

$(2 + 3i)/13 =$ **$2/13 + 3i/13$**.

Q4. Polar form of $-1 + i$.

$r = \sqrt{2}$, Q2 $\Rightarrow \theta = \pi - \pi/4 = 3\pi/4 \Rightarrow$ **$\sqrt{2}(\cos 3\pi/4 + i \sin 3\pi/4)$**.

Q5. Solve $x^2 + x + 1 = 0$.

$D = -3 \Rightarrow x = (-1 \pm i\sqrt{3})/2$.

10 Common mistakes

- $\sqrt{(-4)}\cdot\sqrt{(-9)} = \sqrt{36} = 6$ (wrong; it is -6).
- Taking $\arg z = \tan^{-1}(y/x)$ without checking the quadrant.
- Saying $3 + 2i > 1 + i$ (complex numbers are not ordered).
- Writing the conjugate of $3 - 4i$ as $-3 + 4i$ (it's $3 + 4i$).

11 Practice questions

1. Value of $1 + i^2 + i^4 + i^6 + \ldots + i^{20}$?
2. Find $|(1 + i)/(1 - i)|$.
3. Find real x, y if $(x + iy)(2 - 3i) = 4 + i$.
4. Principal argument of $-\sqrt{3} - i$?
5. Solve $2x^2 + x + 1 = 0$.

ANSWERS

1) 1 · 2) 1 · 3) $x = 5/13, y = 14/13$ · 4) $-5\pi/6$ · 5) $(-1 \pm i\sqrt{7})/4$

12 Quick revision: 5 minutes before the exam

POWERS OF I

- i, −1, −i, 1 cycle.
- Sum of 4 consecutive = 0.

ALGEBRA

- $z\,\bar{z} = |z|^2$.
- $1/z = \bar{z}/|z|^2$.
- $|z_1 z_2| = |z_1||z_2|$.

POLAR

- $r = |z|$, θ in $(-\pi, \pi]$.
- Check quadrant for θ.

QUADRATIC

- $D < 0 \Rightarrow$ conjugate roots.
- Sum -b/a, product c/a.

Have a doubt? Ask the AI Doubt Solver at padhaiadda.netlify.app, or take the mock test on this chapter.