FREE NOTES · NCERT + JEE LEVEL

Trigonometric Functions

Degree aur radian, kisi bhi angle ke trigonometric functions, quadrants me signs, domain aur range, allied angles, sum aur difference formulas, multiple aur sub-multiple angles, product-sum formulas aur general solutions.

01 Degree & radian measure
02 Kisi bhi angle ke trigonometric functions
03 Signs, domain & range
04 Allied angles
05 Sum & difference formulas
06 Multiple & sub-multiple angles
07 Product ↔ sum formulas
08 JEE Extra + Tricks
09 Solved examples
10 Common mistakes
11 Formula sheet & practice
12 Quick revision: exam se 5 minute pehle

01 Degree & radian measure

$$\pi \text{ radian} = 180^\circ$$

$1 \text{ rad} = 180^\circ/\pi \approx 57^\circ 16'; \; 1^\circ = \pi/180 \text{ rad} \approx 0.01746 \text{ rad}$

Arc length $l = r\theta$ (θ radian me)

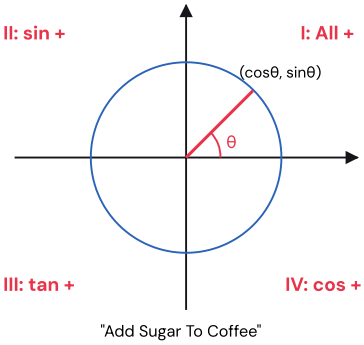
1° = 60′ (minutes), 1′ = 60″ (seconds). Positive angles anticlockwise.

02 Kisi bhi angle ke trigonometric functions

Unit circle pe θ angle wala point (cos θ, sin θ) hai. Isliye **cos²θ + sin²θ = 1**.

- tan θ = sin θ/cos θ, cot θ = 1/tan θ, sec θ = 1/cos θ, cosec θ = 1/sin θ.
- 1 + tan²θ = sec²θ; 1 + cot²θ = cosec²θ.
- sin aur cos har 2π pe repeat (period 2π); tan aur cot har π pe.

θ	0	π/6	π/4	π/3	π/2
sin	0	½	1/√2	√3/2	1
cos	1	√3/2	1/√2	½	0
tan	0	1/√3	1	√3	∞



Unit circle aur har quadrant ke signs

03 Signs, domain & range

Function	Domain	Range
sin x	R	[−1, 1]
cos x	R	[−1, 1]
tan x	R − {(2n+1)π/2}	R
cot x	R − {nπ}	R
sec x	R − {(2n+1)π/2}	R − (−1, 1)
cosec x	R − {nπ}	R − (−1, 1)

Signs (ASTC): I sab positive, II sin/cosec, III tan/cot, IV cos/sec. sin(−x) = −sin x (odd); cos(−x) = cos x (even).

04 Allied angles

RULE

(nπ/2 ± θ) ke liye: n **odd** (π/2, 3π/2) ho to sin ↔ cos, tan ↔ cot. n **even** (π, 2π) ho to function wahi rehta. Phir us quadrant me original function ka sign lagao (θ ko acute maan ke).

Angle	sin	cos	tan
π/2 − θ	cos θ	sin θ	cot θ
π/2 + θ	cos θ	−sin θ	−cot θ
π − θ	sin θ	−cos θ	−tan θ
π + θ	−sin θ	−cos θ	tan θ
2π − θ	−sin θ	cos θ	−tan θ

05 Sum & difference formulas

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = (\tan A \pm \tan B)/(1 \mp \tan A \tan B)$$

$$\cot(A + B) = (\cot A \cot B - 1)/(\cot B + \cot A)$$

Use: sin 75° = sin(45° + 30°) = (√3 + 1)/(2√2); tan 15° = 2 − √3.

06 Multiple & sub-multiple angles

$\sin 2x = 2\sin x\cos x = 2\tan x/(1+\tan^2 x)$

$\cos 2x = \cos^2 x - \sin^2 x = 2\cos^2 x - 1 = 1 - 2\sin^2 x = (1-\tan^2 x)/(1+\tan^2 x)$

$\tan 2x = 2\tan x/(1-\tan^2 x)$

$\sin 3x = 3\sin x - 4\sin^3 x;\ \cos 3x = 4\cos^3 x - 3\cos x$

$\tan 3x = (3\tan x - \tan^3 x)/(1-3\tan^2 x)$

$1+\cos 2x = 2\cos^2 x;\ 1-\cos 2x = 2\sin^2 x$

07 Product ↔ sum formulas

$\sin C + \sin D = 2\sin((C+D)/2)\cos((C-D)/2)$

$\sin C - \sin D = 2\cos((C+D)/2)\sin((C-D)/2)$

$\cos C + \cos D = 2\cos((C+D)/2)\cos((C-D)/2)$

$\cos C - \cos D = -2\sin((C+D)/2)\sin((C-D)/2)$

$2\sin A\cos B = \sin(A+B) + \sin(A-B);\ 2\cos A\cos B = \cos(A+B) + \cos(A-B)$

$2\sin A\sin B = \cos(A-B) - \cos(A+B)$

GENERAL SOLUTIONS

$\sin x = 0 \Rightarrow x = n\pi$; $\cos x = 0 \Rightarrow x = (2n+1)\pi/2$; $\sin x = \sin y \Rightarrow x = n\pi + (-1)^n y$; $\cos x = \cos y \Rightarrow x = 2n\pi \pm y$; $\tan x = \tan y \Rightarrow x = n\pi + y$ $(n \in Z)$.

08 JEE Extra + Tricks

JEE LEVEL

- a sinx + b cosx ki range: $[-\sqrt{(a^2 + b^2)}, \sqrt{(a^2 + b^2)}]$.
- $\sin x \cdot \sin(60° - x) \cdot \sin(60° + x) = \frac{1}{4} \sin 3x$ (cos ke liye cos3x; tan ke liye tan3x).
- $\cos A \cdot \cos 2A \cdot \cos 4A \ldots \cos 2^{n-1}A = \sin(2^n A)/(2^n \sin A)$.
- AP me sines ka jod: $\Sigma \sin(a + rd) = [\sin(nd/2)/\sin(d/2)] \cdot \sin(a + (n - 1)d/2)$.
- General solutions: $\sin\theta = \sin\alpha \Rightarrow \theta = n\pi + (-1)^n\alpha$; $\cos\theta = \cos\alpha \Rightarrow 2n\pi \pm \alpha$; $\tan\theta = \tan\alpha \Rightarrow n\pi + \alpha$.
- Signs ke liye ASTC (All Silver Tea Cups); values: $\sin 18° = (\sqrt{5} - 1)/4$, $\cos 36° = (\sqrt{5} + 1)/4$, $\tan 15° = 2 - \sqrt{3}$.

09 Solved examples

Q1. 40°20′ ko radian me badlo.

40⅓° × π/180 = **121π/540** rad.

Q2. sin(31π/3) nikalo.

31π/3 = 10π + π/3 ⇒ sin(π/3) = **√3/2**.

Q3. cos x = −3/5 aur x quadrant III me, baaki functions nikalo.

sin x = −4/5, tan x = 4/3, cot x = 3/4, sec x = −5/3, cosec x = −5/4.

Q4. Prove karo (sin 5x + sin 3x)/(cos 5x + cos 3x) = tan 4x.

= 2 sin 4x cos x / (2 cos 4x cos x) = tan 4x. ✓

Q5. sin x = −√3/2 solve karo.

sin x = sin(4π/3) ⇒ x = nπ + (−1)^n(4π/3), n ∈ Z.

10 Common mistakes

- l = rθ me degree lagana.
- Allied angles ke baad sign galat (quadrant check karo).
- cos(A + B) = cos A cos B + sin A sin B likhna (minus hota hai).
- General solution me sirf ek angle dena.

11 Formula sheet & practice

Identity	$\sin^2x + \cos^2x = 1$
Double	$\sin 2x = 2\sin x\cos x;\ \cos 2x = 1 - 2\sin^2x$
Triple	$\sin 3x = 3\sin x - 4\sin^3x$
Sum → product	$\sin C + \sin D = 2\sin((C+D)/2)\cos((C-D)/2)$
General	$\sin x = \sin y \Rightarrow x = n\pi + (-1)^n y$

1. cos(−1710°) ki value?
2. tan 15° nikalo.
3. Prove: cos²x + cos²(x + π/3) + cos²(x − π/3) = 3/2.

4. 3 sin x + 4 cos x ki range?

ANSWERS

1) 0 · 2) 2 − √3 · 3) $2\cos^2 A = 1 + \cos 2A$ use karo · 4) [−5, 5]

12 Quick revision: exam se 5 minute pehle

BASICS

- π rad = 180°; $l = r\theta$.
- ASTC signs.
- sin odd, cos even.

COMPOUND

- sin(A±B), cos(A±B) (sign palat-ta).
- tan(A+B) = (tanA+tanB)/(1−tanA tanB).

MULTIPLE

- cos 2x ke 3 roop.
- sin 3x = $3s - 4s^3$; cos 3x = $4c^3 - 3c$.

SOLUTIONS

- sin: $n\pi + (-1)^n y$.
- cos: $2n\pi \pm y$.
- tan: $n\pi + y$.

Koi doubt? padhaiadda.netlify.app pe AI Doubt Solver se poocho, ya is chapter ka mock test do.