FREE NOTES · NCERT + JEE LEVEL

Trigonometric Functions

Angles in degrees and radians, trigonometric functions of any angle, signs in quadrants, domain and range, allied angles, sum and difference formulas, multiple and sub-multiple angles, product-sum formulas and general solutions.

01 Degree & radian measure
02 Trigonometric functions of any angle
03 Signs, domain & range
04 Allied angles
05 Sum & difference formulas
06 Multiple & sub-multiple angles
07 Product ↔ sum formulas
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01 Degree & radian measure

$$\pi \text{ radian} = 180^\circ$$

$1 \text{ rad} = 180^\circ/\pi \approx 57^\circ 16'; \; 1^\circ = \pi/180 \text{ rad} \approx 0.01746 \text{ rad}$

Arc length $l = r\theta$ (θ in radians)

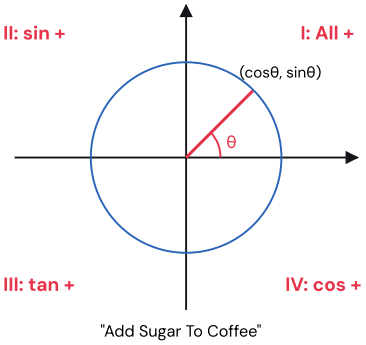
1° = 60′ (minutes), 1′ = 60″ (seconds). Positive angles are anticlockwise.

02 Trigonometric functions of any angle

On a unit circle, a point at angle θ is (cos θ, sin θ).
So **cos²θ + sin²θ = 1**.

- tan θ = sin θ/cos θ, cot θ = 1/tan θ, sec θ = 1/cos θ, cosec θ = 1/sin θ.
- 1 + tan²θ = sec²θ; 1 + cot²θ = cosec²θ.
- sin and cos repeat every 2π (period 2π); tan and cot every π.

θ	0	π/6	π/4	π/3	π/2
sin	0	½	1/√2	√3/2	1
cos	1	√3/2	1/√2	½	0
tan	0	1/√3	1	√3	∞



Unit circle and the signs in each quadrant

03 Signs, domain & range

Function	Domain	Range
sin x	R	[−1, 1]
cos x	R	[−1, 1]
tan x	R − {(2n+1)π/2}	R
cot x	R − {nπ}	R
sec x	R − {(2n+1)π/2}	R − (−1, 1)
cosec x	R − {nπ}	R − (−1, 1)

Signs (ASTC): I all positive, II sin/cosec, III tan/cot, IV cos/sec. sin(−x) = −sin x (odd); cos(−x) = cos x (even).

04 Allied angles

RULE

For (nπ/2 ± θ): if n is **odd** (π/2, 3π/2), sin ↔ cos, tan ↔ cot. If n is **even** (π, 2π), the function stays. Then put the sign of the original function in that quadrant (θ treated as acute).

Angle	sin	cos	tan
π/2 − θ	cos θ	sin θ	cot θ
π/2 + θ	cos θ	−sin θ	−cot θ
π − θ	sin θ	−cos θ	−tan θ
π + θ	−sin θ	−cos θ	tan θ
2π − θ	−sin θ	cos θ	−tan θ

05 Sum & difference formulas

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = (\tan A \pm \tan B)/(1 \mp \tan A \tan B)$$

$$\cot(A + B) = (\cot A \cot B - 1)/(\cot B + \cot A)$$

Use: sin 75° = sin(45° + 30°) = (√3 + 1)/(2√2); tan 15° = 2 − √3.

06 Multiple & sub-multiple angles

$\sin 2x = 2\sin x\cos x = 2\tan x/(1+\tan^2 x)$

$\cos 2x = \cos^2 x - \sin^2 x = 2\cos^2 x - 1 = 1 - 2\sin^2 x = (1-\tan^2 x)/(1+\tan^2 x)$

$\tan 2x = 2\tan x/(1-\tan^2 x)$

$\sin 3x = 3\sin x - 4\sin^3 x;\ \cos 3x = 4\cos^3 x - 3\cos x$

$\tan 3x = (3\tan x - \tan^3 x)/(1-3\tan^2 x)$

$1+\cos 2x = 2\cos^2 x;\ 1-\cos 2x = 2\sin^2 x$

07 Product ↔ sum formulas

$\sin C + \sin D = 2\sin((C+D)/2)\cos((C-D)/2)$

$\sin C - \sin D = 2\cos((C+D)/2)\sin((C-D)/2)$

$\cos C + \cos D = 2\cos((C+D)/2)\cos((C-D)/2)$

$\cos C - \cos D = -2\sin((C+D)/2)\sin((C-D)/2)$

$2\sin A\cos B = \sin(A+B) + \sin(A-B);\ 2\cos A\cos B = \cos(A+B) + \cos(A-B)$

$2\sin A\sin B = \cos(A-B) - \cos(A+B)$

GENERAL SOLUTIONS

$\sin x = 0 \Rightarrow x = n\pi$; $\cos x = 0 \Rightarrow x = (2n+1)\pi/2$; $\sin x = \sin y \Rightarrow x = n\pi + (-1)^n y$; $\cos x = \cos y \Rightarrow x = 2n\pi \pm y$; $\tan x = \tan y \Rightarrow x = n\pi + y$ ($n \in Z$).

08 JEE Extra + Tricks

JEE LEVEL

- Range of a sinx + b cosx: $[-\sqrt{(a^2 + b^2)}, \sqrt{(a^2 + b^2)}]$.
- $\sin x \cdot \sin(60° - x) \cdot \sin(60° + x) = \frac{1}{4} \sin 3x$ (same for cos with cos3x; tan with tan3x).
- $\cos A \cdot \cos 2A \cdot \cos 4A \ldots \cos 2^{n-1}A = \sin(2^n A)/(2^n \sin A)$.
- Sum of sines in AP: $\Sigma \sin(a + rd) = [\sin(nd/2)/\sin(d/2)] \cdot \sin(a + (n - 1)d/2)$.
- General solutions: $\sin\theta = \sin\alpha \Rightarrow \theta = n\pi + (-1)^n\alpha$; $\cos\theta = \cos\alpha \Rightarrow 2n\pi \pm \alpha$; $\tan\theta = \tan\alpha \Rightarrow n\pi + \alpha$.
- ASTC (All Silver Tea Cups) for signs; values: $\sin 18° = (\sqrt{5} - 1)/4$, $\cos 36° = (\sqrt{5} + 1)/4$, $\tan 15° = 2 - \sqrt{3}$.

09 Solved examples

Q1. Convert 40°20′ to radians.

40⅓° × π/180 = **121π/540** rad.

Q2. Find sin(31π/3).

31π/3 = 10π + π/3 ⇒ sin(π/3) = **√3/2**.

Q3. If cos x = −3/5 and x is in quadrant III, find the other functions.

sin x = −4/5, tan x = 4/3, cot x = 3/4, sec x = −5/3, cosec x = −5/4.

Q4. Prove (sin 5x + sin 3x)/(cos 5x + cos 3x) = tan 4x.

= 2 sin 4x cos x / (2 cos 4x cos x) = tan 4x. ✓

Q5. Solve sin x = −√3/2.

sin x = sin(4π/3) ⇒ x = nπ + (−1)^n(4π/3), n ∈ Z.

10 Common mistakes

- Using degrees in l = rθ.
- Getting the sign wrong after using allied angles (check quadrant).
- Writing cos(A + B) = cos A cos B + sin A sin B (it's minus).
- Giving only one angle as the general solution.

11 Formula sheet & practice

Identity	$\sin^2x + \cos^2x = 1$
Double	$\sin 2x = 2\sin x \cos x;\ \cos 2x = 1 - 2\sin^2x$
Triple	$\sin 3x = 3\sin x - 4\sin^3x$
Sum → product	$\sin C + \sin D = 2\sin((C+D)/2)\cos((C-D)/2)$
General	$\sin x = \sin y \Rightarrow x = n\pi + (-1)^n y$

1. Value of cos(−1710°)?
2. Find tan 15°.
3. Prove cos²x + cos²(x + π/3) + cos²(x − π/3) = 3/2.

4. Range of 3 sin x + 4 cos x?

ANSWERS

1) 0 · 2) 2 − √3 · 3) Use $2\cos^2 A = 1 + \cos 2A$ · 4) [−5, 5]

12 Quick revision: 5 minutes before the exam

BASICS

- π rad = 180°; $l = r\theta$.
- ASTC signs.
- sin odd, cos even.

COMPOUND

- sin(A±B), cos(A±B) (sign flips).
- tan(A+B) = (tanA+tanB)/(1−tanA tanB).

MULTIPLE

- cos 2x has 3 forms.
- sin 3x = 3s − 4s³; cos 3x = 4c³ − 3c.

SOLUTIONS

- sin: $n\pi + (-1)^n y$.
- cos: $2n\pi \pm y$.
- tan: $n\pi + y$.

Have a doubt? Ask the AI Doubt Solver at padhaiadda.netlify.app, or take the mock test on this chapter.