FREE NOTES · NCERT + JEE LEVEL

Relations & Functions

Ordered pairs aur Cartesian product, relations (domain, codomain, range), functions, standard real functions aur unke graphs, aur functions ka algebra.

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01 Ordered pairs & Cartesian product

- (a, b) ordered pair hai: (a, b) = (c, d) $\Leftrightarrow$ a = c aur b = d. (1, 2) $\neq$ (2, 1).
- **Cartesian product** $A \times B = \{(a, b) : a \in A, b \in B\}$.
- $n(A \times B) = n(A) \times n(B)$. A ya B empty ho to $A \times B = \varphi$. Aam taur pe $A \times B \neq B \times A$.
- $R \times R = R^2$ yaani xy-plane.

02 Relations

A se B tak **relation** R, $A \times B$ ka koi bhi subset hai. $(a, b) \in R$ ho to a R b likhte; b, a ki **image** hai.

- **Domain** = pehle elements ka set; **codomain** = B; **range** = doosre elements ka set (range $\subseteq$ codomain).
- A se B tak relations ki ginti = 2^{mn} (m = n(A), n = n(B)).
- Roster form, set-builder form ya arrow diagram se dikha sakte.

03 Functions

Function f : A → B aisa relation jisme **A ke har element ki exactly ek image** B me ho.

JALDI CHECK

Arrow diagram: A ke har element se exactly ek arrow. Graph: har vertical line graph ko zyada se zyada ek baar kaate (**vertical line test**). Ordered pairs: koi pehla element alag doosre element ke saath repeat na ho.

{(1,2), (2,3), (3,2)} function hai; {(1,2), (1,3)} nahi. A (m elements) se B (n elements) tak functions = n^m.

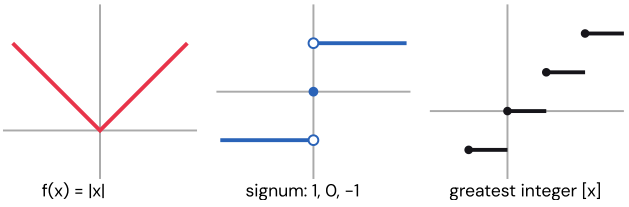
04 Domain & range nikalna

- **Domain:** saare x jinke liye f(x) real number ho. Bacho: zero se bhaag, negative number ka even root, non-positive ka log.
- **Range:** y = f(x) likho, x nikalo, phir dekho kaunse y se real x milta.

f(x)	Domain	Range
1/(x − 2)	R − {2}	R − {0}
√(9 − x²)	[−3, 3]	[0, 3]
x² + 1	R	[1, ∞)
(x − 1)/(x − 3)	R − {3}	R − {1}

05 Standard real functions & graphs

Function	Rule	Domain → Range
Identity	f(x) = x	R → R
Constant	f(x) = c	R → {c}
Polynomial	$a_0 + a_1x + \ldots + a_nx^n$	R
Rational	p(x)/q(x), q(x) ≠ 0	R − {q ke zeros}
Modulus	|x| = x (x ≥ 0), −x (x < 0)	R → [0, ∞)
Signum	x ≠ 0 pe |x|/x, x = 0 pe 0	R → {−1, 0, 1}
Greatest integer	[x] = x se chhota ya barabar sabse bada integer	R → Z



Modulus, signum aur greatest integer functions ke graphs

[2.7] = 2, [−2.7] = −3, [5] = 5.

06 Real functions ka algebra

$(f \pm g)(x) = f(x) \pm g(x)$

$(fg)(x) = f(x)\cdot g(x);\ \ (\alpha f)(x) = \alpha f(x)$

$(f/g)(x) = f(x)/g(x),\ g(x) \neq 0$

f ± g, fg ka domain = $D_f \cap D_g$; f/g ke liye jahan g(x) = 0 wo points bhi hatao.

07 Formula sheet

n(A × B)	$n(A)\cdot n(B)$
Relations A → B	$2^{n(A)\cdot n(B)}$
Functions A → B	$n(B)^{n(A)}$
f ± g ka domain	$D_f \cap D_g$
[x]	x se chhota ya barabar sabse bada integer

08 JEE Extra + Tricks

JEE LEVEL

- A (m) se B (n) tak functions: n^m; one-one ($m \le n$): ${}^nP_m$; onto ($m \ge n$): $\Sigma(-1)^r\ {}^nC_r\ (n - r)^m$.
- Domain tricks: $\sqrt{f} \Rightarrow f \ge 0$; $1/f \Rightarrow f \ne 0$; $\log f \Rightarrow f > 0$; $\sin^{-1}f$, $\cos^{-1}f \Rightarrow -1 \le f \le 1$.
- Greatest integer [x]: $[x + n] = [x] + n$; $\{x\} = x - [x] \in [0, 1)$.
- Even $f(-x) = f(x)$ (y-axis symmetric); odd $f(-x) = -f(x)$ (origin). Har function = even part + odd part.
- Period: sin, cos 2π; tan π; $|\sin x|$ π; $\sin^4x + \cos^4x$ ka $\pi/2$.

09 Solved examples

Q1. A = {1, 2}, B = {3, 4, 5}. $n(A \times B)$ aur A se B tak relations ki ginti?

$n(A \times B) = 6$; relations = 2^6 = **64**.

Q2. $(x + 1, y - 2) = (3, 1)$ ho to x aur y?

$x = 2, y = 3$.

Q3. $f(x) = 2 - 3x$, $x \in R$, $x > 0$ ki range?

$x > 0 \Rightarrow -3x < 0 \Rightarrow f(x) < 2 \Rightarrow$ range = **$(-\infty, 2)$**.

Q4. $f(x) = x^2$, $g(x) = 2x + 1$. $(f + g)(x)$, $(fg)(x)$, $(f/g)(x)$ nikalo.

$x^2 + 2x + 1$; $2x^3 + x^2$; $x^2/(2x + 1)$, $x \ne -½$.

10 Common mistakes

- Sochna ki A × B = B × A.
- Relation ko function kehna jab A ke ek element ki do images hon, ya koi image na ho.
- [−2.3] = −2 likhna (−3 hota hai).
- f/g ke domain se g(x) ke zeros na hatana.

11 Practice questions

1. n(A) = 3, n(B) = 2 ho to A se B tak kitne functions?
2. √(x − 4) ka domain?
3. |x − 2| ki range?
4. Kya {(1,1), (2,1), (3,1)} function hai?
5. [−0.5] + [0.5] ki value?

ANSWERS

1) 8 · 2) [4, ∞) · 3) [0, ∞) · 4) Haan (many-one) · 5) −1

12 Quick revision: exam se 5 minute pehle

PRODUCT & RELATION

- n(A×B) = n(A)n(B).
- Relation = A×B ka subset.
- 2^{mn} relations.

FUNCTION

- Har x: exactly ek image.
- Vertical line test.
- n^m functions.

DOMAIN/RANGE

- ÷0 nahi, √(−) nahi, log(≤0) nahi.
- Range: y = f(x) se x nikalo.

STANDARD

- |x| → [0, ∞).
- signum → {−1, 0, 1}.
- [x] → Z.

Koi doubt? padhaiadda.netlify.app pe AI Doubt Solver se poocho, ya is chapter ka mock test do.