FREE NOTES · NCERT + JEE LEVEL

Relations & Functions

Ordered pairs and Cartesian product, relations (domain, codomain, range), functions, standard real functions and their graphs, and algebra of functions.

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01 Ordered pairs & Cartesian product

- (a, b) is an ordered pair: $(a, b) = (c, d) \iff a = c$ and $b = d$. $(1, 2) \neq (2, 1)$.
- **Cartesian product** $A \times B = \{(a, b) : a \in A, b \in B\}$.
- $n(A \times B) = n(A) \times n(B)$. If A or B is empty, $A \times B = \varphi$. $A \times B \neq B \times A$ in general.
- $R \times R = R^2$ is the xy-plane.

02 Relations

A **relation** R from A to B is any subset of $A \times B$. If $(a, b) \in R$ we write $a \, R \, b$; b is the **image** of a.

- **Domain** = set of first elements; **codomain** = B; **range** = set of second elements (range $\subseteq$ codomain).
- Number of relations from A to B = $\mathbf{2^{mn}}$ ($m = n(A)$, $n = n(B)$).
- Can be shown by roster form, set-builder form or an arrow diagram.

03 Functions

A **function** f : A → B is a relation in which **every element of A has exactly one image** in B.

QUICK TESTS

Arrow diagram: every element of A has exactly one arrow. Graph: every vertical line cuts the graph at most once (**vertical line test**). Ordered pairs: no first element repeats with different second elements.

{(1,2), (2,3), (3,2)} is a function; {(1,2), (1,3)} is not. Number of functions from A (m elements) to B (n elements) = n^m.

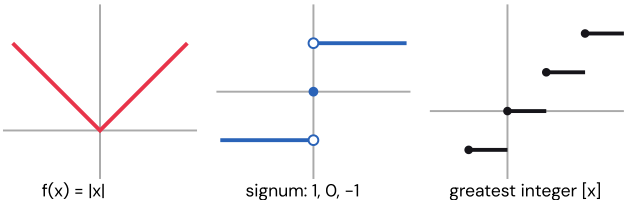
04 Finding domain & range

- **Domain:** all x for which f(x) is a real number. Avoid: division by zero, even root of a negative number, log of a non-positive number.
- **Range:** write y = f(x), solve for x, then find which y give real x.

f(x)	Domain	Range
1/(x − 2)	R − {2}	R − {0}
√(9 − x²)	[−3, 3]	[0, 3]
x² + 1	R	[1, ∞)
(x − 1)/(x − 3)	R − {3}	R − {1}

05 Standard real functions & graphs

Function	Rule	Domain → Range
Identity	f(x) = x	R → R
Constant	f(x) = c	R → {c}
Polynomial	$a_0 + a_1x + \ldots + a_nx^n$	R
Rational	p(x)/q(x), q(x) ≠ 0	R − {zeros of q}
Modulus	|x| = x (x ≥ 0), −x (x < 0)	R → [0, ∞)
Signum	|x|/x for x ≠ 0, 0 at x = 0	R → {−1, 0, 1}
Greatest integer	[x] = greatest integer ≤ x	R → Z



Graphs of modulus, signum and greatest integer functions

[2.7] = 2, [−2.7] = −3, [5] = 5.

06 Algebra of real functions

$(f \pm g)(x) = f(x) \pm g(x)$

$(fg)(x) = f(x)\cdot g(x);\ (\alpha f)(x) = \alpha f(x)$

$(f/g)(x) = f(x)/g(x),\ g(x) \neq 0$

Domain of f ± g, fg = $D_f \cap D_g$; for f/g also remove points where g(x) = 0.

07 Formula sheet

n(A × B)	$n(A)\cdot n(B)$
Relations A → B	$2^{n(A)\cdot n(B)}$
Functions A → B	$n(B)^{n(A)}$
Domain of f ± g	$D_f \cap D_g$
[x]	greatest integer $\leq x$

08 JEE Extra + Tricks

JEE LEVEL

- Number of functions from A (m elements) to B (n): n^m; one-one ($m \le n$): ${}^nP_m$; onto ($m \ge n$): $\Sigma(-1)^r {}^nC_r (n - r)^m$.
- Domain tricks: $\sqrt{f} \Rightarrow f \ge 0$; $1/f \Rightarrow f \ne 0$; $\log f \Rightarrow f > 0$; $\sin^{-1} f$, $\cos^{-1} f \Rightarrow -1 \le f \le 1$.
- Greatest integer $[x]$: $[x + n] = [x] + n$; $\{x\} = x - [x] \in [0, 1)$.
- Even $f(-x) = f(x)$ (symmetric about y-axis); odd $f(-x) = -f(x)$ (about origin). Every function = even part + odd part.
- Period: sin, cos 2π; tan π; $|\sin x|$ π; $\sin^4 x + \cos^4 x$ is $\pi/2$.

09 Solved examples

Q1. $A = \{1, 2\}$, $B = \{3, 4, 5\}$. Find $n(A \times B)$ and the number of relations from A to B.

$n(A \times B) = 6$; relations = 2^6 = **64**.

Q2. If $(x + 1, y - 2) = (3, 1)$, find x and y.

$x = 2$, $y = 3$.

Q3. Find the range of $f(x) = 2 - 3x$, $x \in R$, $x > 0$.

$x > 0 \Rightarrow -3x < 0 \Rightarrow f(x) < 2 \Rightarrow$ range = **$(-\infty, 2)$**.

Q4. $f(x) = x^2$, $g(x) = 2x + 1$. Find $(f + g)(x)$, $(fg)(x)$, $(f/g)(x)$.

$x^2 + 2x + 1$; $2x^3 + x^2$; $x^2/(2x + 1)$, $x \ne -\frac{1}{2}$.

10 Common mistakes

- Thinking A × B = B × A.
- Calling a relation a function when one element of A has two images, or has no image.
- Writing [−2.3] = −2 (it is −3).
- Forgetting to remove zeros of g(x) from the domain of f/g.

11 Practice questions

1. If n(A) = 3, n(B) = 2, how many functions from A to B?
2. Domain of √(x − 4)?
3. Range of |x − 2|?
4. Is {(1,1), (2,1), (3,1)} a function?
5. Value of [−0.5] + [0.5]?

ANSWERS

1) 8 · 2) [4, ∞) · 3) [0, ∞) · 4) Yes (many-one) · 5) −1

12 Quick revision: 5 minutes before the exam

PRODUCT & RELATION

- n(A×B) = n(A)n(B).
- Relation = subset of A×B.
- 2^{mn} relations.

FUNCTION

- Every x: exactly one image.
- Vertical line test.
- n^m functions.

DOMAIN/RANGE

- No ÷0, no √(−), no log(≤0).
- Range: solve y = f(x) for x.

STANDARD

- |x| → [0, ∞).
- signum → {−1, 0, 1}.
- [x] → Z.

Have a doubt? Ask the AI Doubt Solver at padhaiadda.netlify.app, or take the mock test on this chapter.