FREE NOTES · NCERT + JEE LEVEL

Sets

What a set is, roster and set-builder forms, types of sets, subsets and power set, intervals, universal set, Venn diagrams, operations on sets, complement, De Morgan's laws and counting problems.

01 Sets & their representation
02 Types of sets
03 Subsets & power set
04 Intervals (subsets of R)
05 Venn diagrams & operations
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01 Sets & their representation

A **set** is a well-defined collection of distinct objects. "Well-defined" means we can always say whether an object belongs or not. "Good students of a class" is not a set; "students who scored above 90" is.

- Elements: $a \in A$ (belongs), $b \notin A$ (doesn't belong).
- **Roster form:** list the elements: $A = \{1, 2, 3, 4\}$. Order and repetition don't matter.
- **Set-builder form:** state the rule: $A = \{x : x \in N, x < 5\}$.
- Standard sets: N (natural), W, Z (integers), Q (rational), R (real), T (irrational).

02 Types of sets

Type	Meaning	Example
Empty (null, void) φ or $\{\ \}$	No element	$\{x : x \in N, 1 < x < 2\}$
Singleton	Exactly one element	$\{0\}$
Finite	Countable number of elements; n(A) = cardinal number	Vowels
Infinite	Not finite	N, points on a line
Equal	Exactly the same elements	$\{1,2,3\} = \{3,1,2\}$
Equivalent	Same number of elements	$\{a,b\}$ and $\{1,2\}$

$\{0\}$ is not empty (it has the element 0). $\varphi \neq \{\varphi\}$.

03 Subsets & power set

- $A \subseteq B$ if every element of A is in B. $A \subset B$ (proper) if also $A \neq B$.
- $\varphi \subseteq$ every set; every set $\subseteq$ itself. $N \subset Z \subset Q \subset R$.
- If $n(A) = m$: number of subsets = $\mathbf{2^m}$; proper subsets = $2^m - 1$.
- **Power set** P(A) = set of all subsets of A. $n(P(A)) = 2^m$. $P(\{1,2\}) = \{\varphi, \{1\}, \{2\}, \{1,2\}\}$.

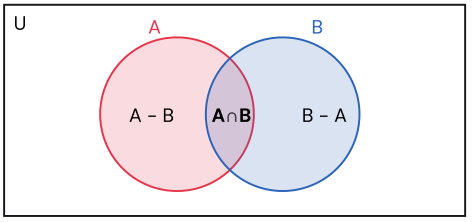
04 Intervals (subsets of R)

Notation	Set	Meaning
(a, b)	$\{x : a < x < b\}$	Open: ends excluded
[a, b]	$\{x : a \leq x \leq b\}$	Closed: ends included
[a, b)	$\{x : a \leq x < b\}$	Half-open
(a, b]	$\{x : a < x \leq b\}$	Half-open

Length of each = b − a. ∞ always gets a round bracket: [2, ∞).

05 Venn diagrams & operations

- **Universal set U:** contains all sets under discussion (rectangle).
- **Union** $A \cup B = \{x : x \in A \text{ or } x \in B\}$.
- **Intersection** $A \cap B = \{x : x \in A \text{ and } x \in B\}$. Disjoint if $A \cap B = \varphi$.
- **Difference** $A - B = \{x : x \in A, x \notin B\}$.
- Symmetric difference $A \Delta B = (A - B) \cup (B - A)$.



Regions of two sets inside U

LAWS

Commutative: $A \cup B = B \cup A$. Associative: $(A \cup B) \cup C = A \cup (B \cup C)$. Identity: $A \cup \varphi = A$, $A \cap U = A$. Idempotent: $A \cup A = A$. Distributive: $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$; $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$.

06 Complement & De Morgan's laws

$$A' = U - A = \{x \in U : x \notin A\}$$

$$(A \cup B)' = A' \cap B' \quad (A \cap B)' = A' \cup B'$$

Also (A′)′ = A, A ∪ A′ = U, A ∩ A′ = φ, U′ = φ, φ′ = U. A − B = A ∩ B′.

07 Counting formulas

$$n(A \cup B) = n(A) + n(B) - n(A \cap B)$$

$$n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B) - n(B \cap C) - n(A \cap C) + n(A \cap B \cap C)$$

$$n(A - B) = n(A) - n(A \cap B); \; n(A') = n(U) - n(A)$$

$$\text{Only } A = n(A) - n(A \cap B) - n(A \cap C) + n(A \cap B \cap C)$$

08 JEE Extra + Tricks

JEE LEVEL

- $n(A \cup B \cup C) = \Sigma n(A) - \Sigma n(A \cap B) + n(A \cap B \cap C)$.
- Number of subsets of a set with n elements = 2^n; proper subsets $2^n - 1$; non-empty proper $2^n - 2$.
- Number of relations from A (m) to B (n) = 2^{mn}.
- Symmetric difference $A \Delta B = (A - B) \cup (B - A)$; $A - B = A \cap B'$.
- Venn trick for "only A": $n(A) - n(A \cap B) - n(A \cap C) + n(A \cap B \cap C)$.

09 Solved examples

Q1. Write {x : x is a prime factor of 30} in roster form.

{2, 3, 5}.

Q2. In a class of 35, 24 like cricket, 16 like football, each likes at least one. How many like both?

$35 = 24 + 16 - x \Rightarrow x =$ **5**.

Q3. If A has 3 elements, find n(P(P(A)))?

$n(P(A)) = 8 \Rightarrow n(P(P(A))) = 2^8 =$ **256**.

Q4. U = {1,...,9}, A = {1,2,3,4}, B = {2,4,6,8}. Verify $(A \cup B)' = A' \cap B'$.

$A \cup B = \{1,2,3,4,6,8\} \Rightarrow (A \cup B)' = \{5,7,9\}$. $A' = \{5,6,7,8,9\}$, $B' = \{1,3,5,7,9\} \Rightarrow A' \cap B' = \{5,7,9\}$. ✓

10 Common mistakes

- Writing {φ} as the empty set (it has one element).
- Repeating elements in roster form or thinking order matters.
- Forgetting to subtract n(A ∩ B) in union counting.
- Using [with ∞.
- Confusing ∈ (element) with ⊂ (subset): 1 ∈ {1,2} but {1} ⊂ {1,2}.

11 Practice questions

1. Is $\{x : x^2 = 4, x \text{ is odd}\}$ empty?
2. Number of proper subsets of {a, b, c, d}?
3. Write (−3, 5] in set-builder form.
4. If n(U) = 50, n(A) = 20, find n(A′).
5. Simplify A ∩ (A ∪ B).

ANSWERS

1) Yes · 2) 15 · 3) $\{x : x \in R, -3 < x \le 5\}$ · 4) 30 · 5) A

12 Quick revision: 5 minutes before the exam

BASICS

- Well-defined collection.
- Roster vs set-builder.
- φ ⊆ every set.

COUNTING

- Subsets 2^n, proper $2^n - 1$.
- n(A∪B) = n(A)+n(B)−n(A∩B).

OPERATIONS

- A − B = A ∩ B′.
- Distributive laws.

DE MORGAN

- (A∪B)′ = A′∩B′.
- (A∩B)′ = A′∪B′.

Have a doubt? Ask the AI Doubt Solver at padhaiadda.netlify.app, or take the mock test on this chapter.